

Edps 590BAY
Fall 2022
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Homework 2
Due September 13, 2022

1. For each of the scenarios, in your opinion would an informative or non informative prior be most appropriate? Briefly explain why you selected the prior.
 - (a) A research group at small eastern private college has done a study and used students at the college as subjects and their work yielded a surprising result. You decide to replicate the study at UIUC and use Bayesian method to analyze your data.
 - (b) You are conducting an experiment using fMRI, which is very expensive. Since there is only one machine available, you collect data from subjects one at a time. To try to minimize your cost and time to complete the project, you decide to only collect data until you obtain a reasonable result within a pre-determined level of precision.
 - (c) You are trying to predict the probability of getting lung cancer given that a person is a smoker. Data from retrospective studies exist that give the incidence of lung cancer in the population of interest, the incidence of smoking in the population, and the incidence of a person being a smoker among lung cancer patients.
2. To learn more about the Beta distribution and how shape depends on the parameters, plot the following Beta densities:
 - (a) Beta(0.5, 0.5)
 - (b) Beta(10.2, 1.5)
 - (c) Beta(1.5, 10.2)
 - (d) Beta(100, 62)

3. To get a better feel for Beta prior, Beta data, and Beta posterior, use the `LearnBayes` package and the `triplot` for the following:
- (a) prior $\theta \sim \text{Beta}(1, 1)$, likelihood $y \sim \text{Binomial}(14, 17)$, and posterior $\theta|y$.
 - i. Plot prior, likelihood and posterior. Comment.
 - ii. What is the mean of the posterior?
 - iii. What is the mode of the posterior?
 - iv. What are the 95% credible intervals (quantiles)?
 - v. What are the 95% high density intervals? (use `HDIinterval` package)
 - (b) ... and for prior $\theta \sim \text{Beta}(5, 10)$, likelihood $y \sim \text{Binomial}(14, 17)$, and posterior $\theta|y$.
 - i. Plot prior, likelihood and posterior. Comment and compare with those from part (a)i.
 - ii. What is the posterior distribution?
 - iii. What is the mean of the posterior?
 - iv. What is the mode of the posterior?
 - v. What are the 95% credible intervals (quantiles)?
 - vi. What are the 95% high density intervals? (use `HDIinterval` package)
4. Below is data from the 2006, 2016 and 2018 General Social Survey where respondents indicated whether they favor or oppose a law which would require a person to obtain a police permit before he or she could buy a gun. Note that these are cross-sectional data (i.e., different respondent each year).

	Response	
Year	favor	oppose
2006	1568	395
2016	1330	528
2018	1102	439

- (a) For the year 2006,
 - i. What is the sample proportion who favor a law?

- ii. What is the posterior distribution for the proportion of people who favor gun control (use uniform prior)? Also, plot the prior, likelihood and posterior and comment on the plot.
 - iii. What is the mean of this distribution?
 - iv. What is the 95% credible interval? Interpret.
 - v. What is the 95% high density interval? Interpret.
 - vi. If you were reporting this result in a paper, which interval would you use?
- (b) For the year 2016,
- i. What is the sample proportion who favor a law?
 - ii. Using data from 2006 as the prior, what is the posterior distribution for the proportion of people who favor gun control? Also plot the prior, likelihood and posterior and comment on the plot.
 - iii. What is the mean of this distribution?
 - iv. What is the 95% credible interval? Interpret.
 - v. What is the 95% high density interval? Interpret.
 - vi. If you were reporting this result in a paper, which interval would you use?
- (c) Are the probabilities favoring a law from 2006 and 2016 the same or different? (i.e., 2 independent samples, Monte Carlo).
5. Two Proportions (optional)
- If you want to give Monte Carlo a try, are the proportions from 2006 and 2016 the same or not. Use non-informative priors for posteriors for both 2006 and 2016. (Why?)